Finding integers from group orbits

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In this project, we study the "$local-global$" principal for integers coming from group orbits. Take a vector $v_0 \in \mathbb{Z}^2$, and Γ a subgroup of $SL_2(\mathbb{Z})$, we are interested in a subset of integers $\mathscr{S} := < w_0, \Gamma \cdot v_0 >$ from the group orbit $\mathscr{O} := \Gamma \cdot v_0$. An easier *admissible* set $\mathscr{A}$ gives all the *local* obstruction $\mathscr{S}$. The "$local-global$" principal says that almost all admissible integers are in $\mathscr{S}$. This question naturally arise from the study of curvatures of integral *Apollonian* gasket or *Zaremba*'s conjecture on denominators of continued fraction expansions.

We investigate the "$local-global$" principal for several examples of $\Gamma < SL_2(\mathbb{Z})$. When the "*cirital exponent*", which measure the growth of the subgroup is not too small, we indeed see "$local-global$" principal holds numerically.